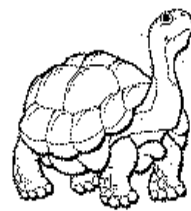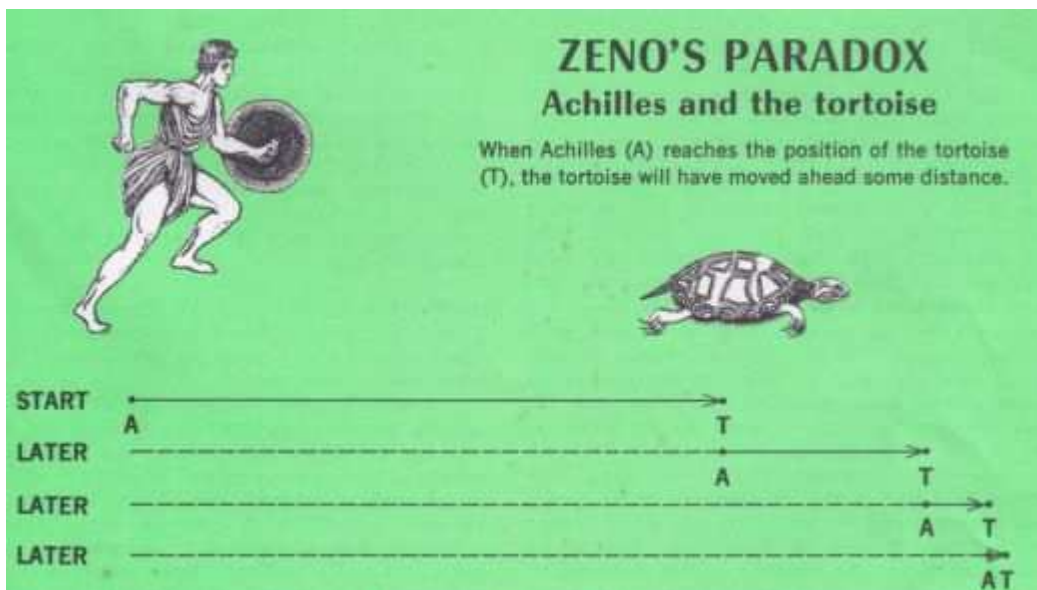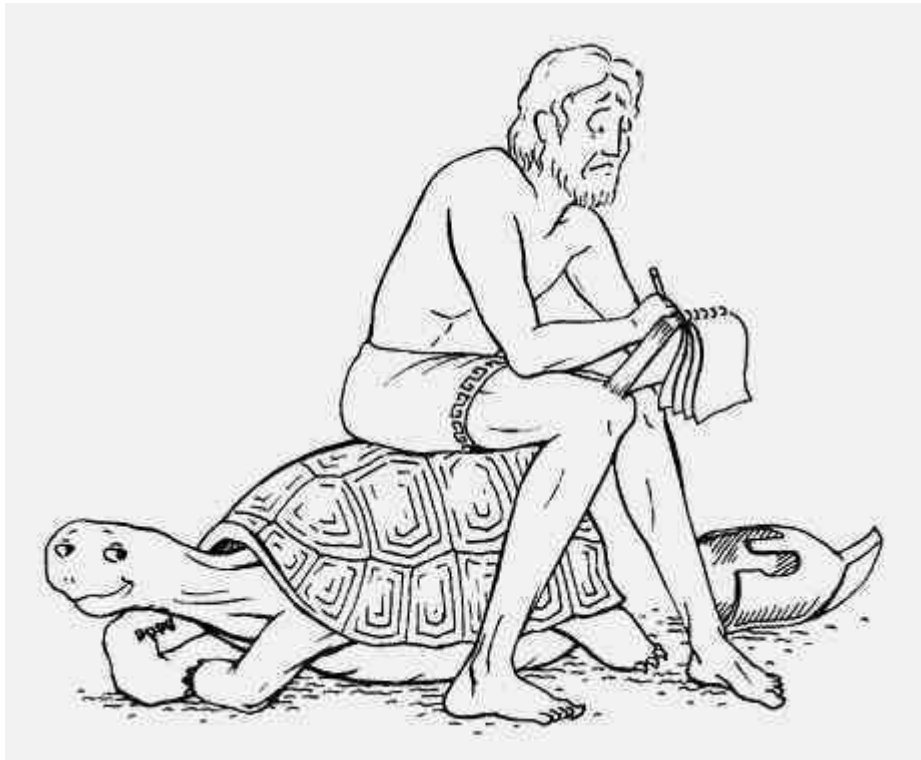(490 - 430 bc.)

Achilles and the Tortoise

Achilles and the Tortoise

2

Zeno proposed a race between Achilles and a tortoise. Aware of AchiDes' speed and the tortoise's slowness, he suggested that, in all fairness, the tortoise be given a slight head start. The result of the race, according to Zeno, is that no matter how long the track, Achilles never even catches up to the tortoise. The reasoning behind this absurd result is that no matter how fast Achilles runs, by the time he reaches the place where the tortoise started, the tortoise will have moved ahead some distance to a new position; by the time Achilles reaches the new position, the tortoise will have crawled ahead again. Thus, Achilles will come closer and closer, but he will never be able to catch up to the tortoise.

ZENO'S PARADOX
Achilles and the tortoise
When Achilles (A) reaches the position of the tortoise (T), the tortoise will have moved ahead some distance.

Analysis of These Paradoxes

There are obvious similarities between these two paradoxes. First, both of them involve absurd conclusions. A sentence certainly cannot be true only on the condition that it be false, and vice versa. Achilles certainly will catch up with the tortoise; indeed, if the speed of each is known, along with the length of the tortoise's head start, the number of minutes it will take him to catch up can be determined precisely. Secondly, it is not immediately clear that there has been any mistake in the reasoning that leads to these conclusions. "This sentence is false" seems to be a perfectly good sentence in English; Achilles will indeed have to reach the spots the tortoise has occupied before he can catch it.

It is this second feature of the paradoxes that differentiates them from a familiar type of argument known as reductio ad absurdum (Latin for "reduction to absurdity"). To prove, for instance, that a man is innocent of some crime, he is assumed guilty, and absurd consequences are then derived from this assumption. Since the assumption has led to absurd results, it must be false and is therefore discarded. But in Zeno's and

Epimenides' paradoxes, the assumptions do not appear to be suspect.

Nevertheless, something must be generating tne absurdity. The difficulty is that in a typical logical paradox, men do not suspect their assumptions or reasonings because they are committed to them. To alter them would involve an uncomfortable change in their ways of talking or thinking. It is in this respect that a paradox is "contrary to opinion." Because the study of paradoxes forces the logician to alter common opinions it exercises an important function.

Despite the similarities between the two paradoxes, it might be argued that Zeno's paradox is actually different from Epimenides' in the second respect. That is, it might be said that Zeno's reasoning contains a hidden flaw, and that his paradox should therefore be more properly called a fallacy. Zeno's argument is so complex that there is to this day a good deal of disagreement as to the precise source of the absurdity. Nevertheless, many feel that the error lies in his assumption that an infinite succession of intervals of time must add up to eternity and that this error forces him to conclude that Achilles will never catch the tortoise. Zeno pictures the relative positions of the runners at ever shorter periods of time. Since there seems to be no end of intervals to choose, Zeno concludes that Achilles never catches up. But it is not necessarily true that an infinite succession of time intervals must add up to eternity. Some infinite series, called convergent series, by definition do not add upto infinity. (The series $1 + 1 + 1 + 1 + 1 \ldots$, for instance, is a convergent series. It has infinitely many terms, yet its sum, far from being infinite, never exceeds 1.) To avoid the paradox, one need merely see that Zeno is wrongly assuming that an infinite series must have an infinite sum.

But with the liar paradox there is no such well-established escape. The best solution offered to date, one not developed until the 20th century, is that the paradox results from confusing two languages: the object language, or language used to talk about objects, and the meta-language, or language used to talk about the object language. The trouble with "This sentence is false" is that it attempts to be in the object language and the meta-language simultaneously. In referring to itself, the term "false" becomes incurably ambiguous; the sentence is thus meaningless.